Λ_b LIFETIME FROM THE HQET SUM RULE

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The HQET sum rule analysis for the Λ_b matrix element of the four-quark operator relevant to its lifetime is reported. Our main conclusion is that the lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$ can be as low as 0.91.

1 Introduction

The experimental result on the lifetime ratio of the Λ_b baryon and B meson, $\tau(\Lambda_b)/\tau(B^0) = 0.795 \pm 0.053$ [1], still needs theoretical understanding. It can be calculated systematically by heavy quark expansion [2], if we do not assume the failure of the local duality assumption. To the order of $1/m_b^2$, the calculated ratio is still close to unity. The potential importance of the $O(1/m_b^3)$ effect has been pointed out [3,4,5]. The lifetime ratio was calculated as follows [3],

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} \simeq 0.98 + \xi\{p_1 B_1(m_b) + p_2 B_2(m_b) + p_3 \epsilon_1(m_b) + p_4 \epsilon_2(m_b) + [p_5 + p_6 \tilde{B}(m_b)] r(m_b)\} \,, \tag{1}$$

where the term proportional to $\xi \equiv (f_B/200\text{MeV})^2$ arises from the $1/m_b^3$ contributions. At the scale m_b, the values of the perturbative coefficients p_i's are $p_1 = -0.003$, $p_2 = 0.004$, $p_3 = -0.173$, $p_4 = -0.195$, $p_5 = -0.012$, $p_6 = -0.021$. B_1, B_2, ϵ_1, ϵ_2, r and $\tilde{B}$ are the parameterization of the hadronic matrix elements of the following four-quark operators,

$$\begin{aligned}
\langle \bar{B}|\bar{b}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma^\mu(1-\gamma_5)b|\bar{B}\rangle &\equiv B_1 f_B^2 m_B^2 \,, \\
\langle \bar{B}|\bar{b}(1-\gamma_5)q\bar{q}(1+\gamma_5)b|\bar{B}\rangle &\equiv B_2 f_B^2 m_B^2 \,, \\
\langle \bar{B}|\bar{b}\gamma_\mu(1-\gamma_5)t_a q\bar{q}\gamma^\mu(1-\gamma_5)t_a b|\bar{B}\rangle &\equiv \epsilon_1 f_B^2 m_B^2 \,, \\
\langle \bar{B}|\bar{b}(1-\gamma_5)t_a q\bar{q}(1+\gamma_5)t_a b|\bar{B}\rangle &\equiv \epsilon_2 f_B^2 m_B^2 \,,
\end{aligned} \tag{2}$$

and

$$\begin{aligned}
\frac{1}{2m_{\Lambda_b}}\langle \Lambda_b|\bar{b}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma^\mu(1-\gamma_5)b|\Lambda_b\rangle &\equiv -\frac{f_B^2 m_B}{12} r \,, \\
\frac{1}{2m_{\Lambda_b}}\langle \Lambda_b|\bar{b}(1-\gamma_5)q\bar{q}(1+\gamma_5)b|\Lambda_b\rangle &\equiv -\tilde{B}\frac{f_B^2 m_B}{24} r \,.
\end{aligned} \tag{3}$$

These parameters have been calculated by QCD sum rules. In Refs. [6,7], the mesonic parameters B_i and ϵ_i were calculated within the framework of heavy quark effective theory (HQET). The baryonic parameters r and $\tilde{B}$ were calculated in Refs. [8] and [9]. Here we report our result of [9].

2 The Calculation

The new ingredients of our analysis compared to Ref. [8] is the following. (1) Gluon condensate and six-quark condensate are included. (2) A different duality assumption is adopted.

The result of $\tilde{B} = 1$ does not change in the valence quark approximation.

To calculate r, the following three-point Green's function is constructed,

$$\Pi(\omega, \omega') = i^2 \int dxdy e^{ik'\cdot x - ik\cdot y} \langle 0|\mathcal{T}\tilde{j}^v(x)\tilde{O}(0)\bar{\tilde{j}}^v(y)|0\rangle \; , \tag{4}$$

where $\omega = v \cdot k$ and $\omega' = v \cdot k'$. The Λ_Q baryonic current $\tilde{j}^v$ is [10,11,12,13],

$$\tilde{j}^v = \epsilon^{abc} q_1^{Ta} C\gamma_5 (a + b \not{v})\tau q_2^b h_v^c \; , \tag{5}$$

where a and b are certain constants, h_v is the heavy quark field in the HQET with velocity v, C is the charge conjugate matrix, τ is the flavor matrix for Λ_Q. In Eq. (4), $\tilde{O}$ denotes the four-quark operator

$$\tilde{O} = \bar{h_v}\gamma_\mu \frac{1-\gamma_5}{2} h_v \bar{q}\gamma^\mu \frac{1-\gamma_5}{2} q \; . \tag{6}$$

Note $< \Lambda_b|\tilde{O}|\Lambda_b >= - < \Lambda_b|O|\Lambda_b >$ where

$$O = \bar{h_v}\gamma_\mu \frac{1-\gamma_5}{2} q \bar{q}\gamma^\mu \frac{1-\gamma_5}{2} h_v \; . \tag{7}$$

In terms of the hadronic expression, the parameter r appears in the ground state contribution of $\Pi(\omega, \omega')$,

$$\Pi(\omega, \omega') = \frac{1}{2} \frac{f_\Lambda^2 \langle \Lambda_Q|O|\Lambda_Q\rangle}{(\bar{\Lambda} - \omega)(\bar{\Lambda} - \omega')} \frac{1+\not{v}}{2} + \text{higher states} \; . \tag{8}$$

$\bar{\Lambda} = m_{\Lambda_Q} - m_Q$ and the quantity f_Λ is defined as $\langle 0|\tilde{j}^v|\Lambda_Q\rangle \equiv f_\Lambda u$ with u being the unit spinor in the HQET. The QCD sum rule calculations for f_Λ were given in Refs. [10,11,12,13].

On the other hand, this Green's function $\Pi(\omega, \omega')$ can be calculated in terms of quark and gluon language with vacuum condensate straightforwardly.

The fixed point gauge is used [14]. The tadpole diagrams in which the light quark lines from the four-quark vertex are contracted have been subtracted. While the calculation can be justified if $(-\omega)$ and $(-\omega')$ are large, however the hadron ground state property should be obtained at small $(-\omega)$ and $(-\omega')$. These contradictory requirements are achieved by introducing double Borel transformation for ω and ω'.

3 Duality Assumption

Generally the duality is to simulate the higher states by the whole quark and gluon contribution above some threshold energy ω_c. The whole contribution of the three-point correlator $\Pi(\omega,\omega')$ can be expressed by the dispersion relation,

$$\Pi(\omega,\omega') = \frac{1}{\pi}\int_0^\infty d\nu \int_0^\infty d\nu' \frac{\mathrm{Im}\Pi(\nu,\nu')}{(\nu-\omega)(\nu'-\omega')} \ . \tag{9}$$

With the redefinition of the integral variables

$$\nu_+ = \frac{\nu+\nu'}{2} \ , \quad \nu_- = \frac{\nu-\nu'}{2} \ , \tag{10}$$

the integration becomes

$$\int_0^\infty d\nu \int_0^\infty d\nu' ... = 2\int_0^\infty d\nu_+ \int_{-\nu_+}^{\nu_+} d\nu_- ... \ . \tag{11}$$

It is in ν_+ that the quark-hadron duality is assumed [15,16],

$$\text{higher states} = \frac{2}{\pi}\int_{\omega_c}^\infty d\nu_+ \int_{-\nu_+}^{\nu_+} d\nu_- \frac{\mathrm{Im}\Pi(\nu,\nu')}{(\nu-\omega)(\nu'-\omega')} \ . \tag{12}$$

This kind of assumption was suggested in calculating the Isgur-Wise function in Ref. [15] and was argued for in Ref. [16].

The sum rule for $\langle\Lambda_Q|\tilde{O}|\Lambda_Q\rangle$ after the integration with the variable ν_- is

$$\begin{aligned}\frac{(a+b)^2}{2} f_\Lambda^2 \exp\left(-\frac{\bar\Lambda}{T}\right)\langle\Lambda_Q|\tilde{O}|\Lambda_Q\rangle = \int_0^{\omega_c} d\nu \exp\left(-\frac{\nu}{T}\right)\{&\frac{a^2+b^2}{840\pi^6}\nu^8 - \frac{ab}{6\pi^4}\nu^5\langle\bar{q}q\rangle \\ &+\frac{3(a^2+b^2)}{2048\pi^6}\nu^4\langle g_s^2G^2\rangle + \frac{5ab}{48\pi^4}m_0^2\langle\bar{q}q\rangle\nu^3 \\ &+\kappa_1\frac{17(a^2+b^2)}{96\pi^2}\langle\bar{q}q\rangle^2\nu^2\} - \kappa_2\frac{ab}{144}\langle\bar{q}q\rangle^3 \ ,\end{aligned}\tag{13}$$

where κ_1, κ_2 are the parameters used to indicate the deviation from the factorization assumption for the four- and six-quark condensates. $\kappa_{1,2} = 1$ corresponds to the vacuum saturation approximation. $\kappa_1 = (3 \sim 8)$ is introduced in order to include the nonfactorizable contribution and to fit the data [17]. There is no discussion of κ_2 in literature so we use $\kappa_2 = 1$. We shall adopt $a = b = 1$ [12] in our numerical analysis. The parameters $f_\Lambda and \bar{\Lambda}$ were obtained by the HQET sum rule analysis of two-point correlator [10,11,12,13].

Our final sum rule is obtained from Eq.(13) by dividing that for f_Λ. The value of ω_c is (1.2 ± 0.1) GeV. The sum rule window is $T = (0.15 - 0.35)$ GeV. We obtain for $\kappa_1 = 4$

$$\langle \Lambda_Q | \tilde{O} | \Lambda_Q \rangle = (1.6 \pm 0.4) \times 10^{-2} \mathrm{GeV}^3 \quad \text{or} \quad r = (3.6 \pm 0.9) \; . \tag{14}$$

By taking $f_B = 200$ MeV. If we use $\kappa_1 = 1$, we get

$$\langle \Lambda_Q | \tilde{O} | \Lambda_Q \rangle = (5.5 \pm 1.0) \times 10^{-3} \mathrm{GeV}^3 \quad \text{or} \quad r = (1.3 \pm 0.3) \; . \tag{15}$$

Note that our results depend on ω_c weakly.

The value of r we have obtained above is at some hadronic scale, because we have been working in the HQET. By choosing $\alpha_s(\mu_{had}) = 0.5$ (corresponding to $\mu_{had} \sim 0.67$ GeV), we obtain $\tilde{B}(m_b) \simeq 0.58$ and

$$r(m_b) \simeq (6.2 \pm 1.6) \;\; \text{for} \;\; \kappa_1 = 4 \; , \quad \text{and} \;\; r(m_b) \simeq (2.3 \pm 0.6) \;\; \text{for} \;\; \kappa_1 = 1 \; . \tag{16}$$

The Λ_b and B^0 lifetime ratio given in Eq. (1) is expressed specifically as

$$\begin{aligned} \frac{\tau(\Lambda_b)}{\tau(B^0)} &\simeq 0.83 \pm 0.04 \;\; \text{for} \;\; \kappa_1 = 4 \; , \\ &\simeq 0.93 \pm 0.02 \;\; \text{for} \;\; \kappa_1 = 1 \; . \end{aligned} \tag{17}$$

Where the values $\epsilon_1(m_b) = -0.08$ and $\epsilon_2(m_b) = -0.01$ have been taken from the QCD sum rules [6]. We see that with the vacuum saturation ($\kappa_1 = 1$), although r is enhanced by about six times compared to that in Ref. [8], it is still not large enough to account for the data. The life time ratio between Λ_b and B mesons can be explained if we also take into account the nonfactorizable contribution of the four-quark condensate.

4 Conclusion

In summary, we have reanalyzed the QCD sum rule for the Λ_b matrix element of the four-quark operator relevant to the lifetime of Λ_b. The difference between Ref. [8] and ours is mainly because of duality assumptions. While a large nonfactorizable effect in the four-quark condensate can make the theoretical result consistent with the experiment, our main conclusion is that the lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$ can be as low as 0.91.

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